Quark-lepton complementarity and self-complementarity in different schemes

Yijia Zhang and Xinyi Zhang
School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

Bo-Qiang Ma*
School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China and Center for High Energy Physics, Peking University, Beijing 100871, China

arXiv:1211.3198v2 [hep-ph] 17 Nov 2012

With the progress of increasingly precise measurements on the neutrino mixing angles, phenomenological relations such as quark-lepton complementarity (QLC) among mixing angles of quarks and leptons and self-complementarity (SC) among lepton mixing angles have been observed. Using the latest global fit results of the quark and lepton mixing angles in the standard Chau-Keung scheme, we calculate the mixing angles and *CP*-violating phases in the other eight different schemes. We check the dependence of these mixing angles on the *CP*-violating phases in different phase schemes. The dependence of QLC and SC relations on the *CP* phase in the other eight schemes is recognized and then analyzed, suggesting that measurements on *CP*-violating phases of the lepton sector are crucial to the explicit forms of QLC and SC in different schemes.

PACS numbers: 14.60.Pq, 11.30.Er, 12.15.Ff, 14.60.Lm

I. INTRODUCTION

After decades of neutrino oscillation experiments, it is generally taken for granted that neutrinos are massive particles that can vary among all the three flavors through the oscillation process described by neutrino mixing. One of the most important issues concerning neutrino mixing is the determination of the neutrino mixing matrix, i.e., the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix [1], which is the lepton sector counterpart of the quark sector mixing matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix [2]. The PMNS matrix is defined as the correlation matrix linking neutrino flavor eigenstates $|\nu_{\rm flavor}\rangle$ and mass eigenstates $|\nu_{\rm mass}\rangle$,

$$|\nu_{\rm mass}\rangle = U_{\rm PMNS}|\nu_{\rm flavor}\rangle. \tag{1}$$

This mixing matrix is conventionally represented in the standard Chau-Keung (CK) scheme [3] as

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}s_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\alpha} & \\ & & e^{i\beta} \end{pmatrix}, \tag{2}$$

where three mixing angles are denoted by θ_{12}, θ_{23}, and θ_{13}, with their trigonometric functions $\sin\theta_{12}$, $\cos\theta_{12}$, etc. represented by s_{12}, c_{12}, etc. respectively. The *CP*-violating phase is denoted by δ, meanwhile α and β represent the other two phases in the case of Majorana neutrinos. In the case of Dirac neutrinos, the latter two phases α and β can be removed by redefinition, thus there remain only four independent parameters, i.e., three mixing angles together with one *CP*-violating phase. If the neutrinos are of Majorana type, the two phases α and β are needed for a full determination of the mixing matrix. As the Majorana phases do not manifest themselves in the oscillation, we ignore these two phases α and β and take only the first term on the right-hand side of Eq. (2) in this article. By now, the quark-sector mixing matrix has been measured with good precision. In the lepton sector, the values of the three mixing angles have been measured after years of neutrino oscillation experiments, though with relatively lower precision compared to the quark case.

The explicit form of the fermion mixing matrix is not unique and an alternative scheme is the Kobayashi-Maskawa (KM) scheme [4]

$$U_{\rm PMNS} = \begin{pmatrix} c_1 & s_1c_3 & -s_1s_3 \\ -s_1c_2 & c_1c_2c_3+s_2s_3e^{-i\delta} & -c_1c_2s_3+s_2c_3e^{-i\delta} \\ s_1s_2 & -c_1s_2c_3+c_2s_3e^{-i\delta} & c_1s_2s_3+c_2c_3e^{-i\delta} \end{pmatrix}. \tag{3}$$

In the KM scheme, as will be mentioned later, the *CP*-violating phase of the quark sector is quite near 90°, leading to the hypothesis of "maximal *CP* violation" [5–10]. Besides the CK and KM schemes, in Ref. [11] all the other possible schemes of the mixing matrix are considered and presented. There are actually 12 schemes of mixing matrix. Among them, 3 schemes can be trans-

formed into others through straightforward redefinition of mixing angles, thus leaving 9 different schemes [11–14], whose forms are provided in Sec. II.

Quark-Lepton Complementarity (QLC) [15–18] and Self-Complementarity (SC) [13, 19] are phenomenological relations of quark and lepton mixing angles. They provide a novel connection to link quarks and leptons. They can be expressed in a more clear way as (4)-(7):

$$\text{QLC1:} \quad \theta_{12} + \vartheta_{12} = 45^\circ, \tag{4}$$
$$\text{QLC2:} \quad \theta_{23} + \vartheta_{23} = 45^\circ, \tag{5}$$
$$\text{SC1:} \quad \vartheta_{12} + \vartheta_{13} = 45^\circ, \tag{6}$$
$$\text{SC2:} \quad \vartheta_{12} + \vartheta_{13} = \vartheta_{23}. \tag{7}$$

(From now on we use ϑ to represent lepton sector mixing angles to distinguish them from quark sector mixing angles θ.)

Here we have marked the two QLC relations by QLC1 and QLC2, and the two slightly different SC relations by SC1 and SC2 respectively. Originally these phenomenological relations are observed only in the CK scheme and fit the experimental results within small errors. However, a question naturally arises, i.e., whether these relations still hold in schemes other than the CK scheme since we cannot find any justification for the priority of the CK scheme. There are already some researches on QLC and SC in the nine schemes [12–14]. However, all of these examinations of QLC and SC are carried out under some fixed phase choices. Since the *CP*-violating phase of the lepton sector is not determined from current experiment, in this article we examine QLC and SC with the whole range variation of the lepton *CP*-violating phase. These will be treated in Sec. III and Sec. IV.

We purpose to make a detailed re-analysis of QLC and SC in all the nine schemes, emphasizing on the influences due to the variation of the lepton *CP*-violating phase. In Sec. II we use the latest experiment results to do calculations on mixing angles and *CP*-violating phases in all the nine schemes. In Sec. III we focus on QLC, examine these complementarity relations and make some analyses. In Sec. IV we similarly examine and analyze SC. In Sec. V we discuss some properties of *CP*-violating phases among different schemes with a suggestion of convention redefinition, and suggest some empirical relations among quark *CP*-violating phases in different schemes.

II. QUARK AND LEPTON MIXING ANGLES AND CP-VIOLATING PHASES

First we list all the nine schemes mentioned in Sec. I in Table I. To avoid ambiguities, the explicit forms of the rotation matrices are provided:

$$R_{12}(\theta_3) = \begin{pmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

$$R_{31}(\theta_3) = \begin{pmatrix} c_3 & 0 & s_3 \\ 0 & 1 & 0 \\ -s_3 & 0 & c_3 \end{pmatrix}, \tag{9}$$

$$R_{12}(\theta_3, \phi) = \begin{pmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & e^{-i\phi} \end{pmatrix}, \tag{10}$$

and the others are similarly defined. From (2), (3) and Table I, P3 is the same as the KM scheme, and P1 is equivalent to the CK scheme. The *CP*-violating phases in Table I are denoted by ϕ, rather than δ, to remind readers of the slight difference between the CK scheme in (2) and the P1 scheme in Table I. Actually, when we identify δ as ϕ, i.e., $\delta = \phi$, we get an equation between the PMNS matrices in the CK scheme and the one in the P1 scheme:

$$U_{\text{CK}} = \begin{pmatrix} 1 & & \\ & e^{i\phi} & \\ & & e^{i\phi} \end{pmatrix} U_{\text{P1}} \begin{pmatrix} 1 & & \\ & 1 & \\ & & e^{-i\phi} \end{pmatrix}. \tag{11}$$

The phase factors in the two matrices in (11) can be eliminated by unphysical phase redefinition of lepton fields in the case of Dirac neutrinos. In the case of Majorana neutrinos, the phase factor in the matrix to the right of U_{P1} can be absorbed by redefinition of Majorana phases, while the phase factors in the matrix to the left of U_{P1} are still eliminated.

A. Quark sector

We begin our quark-sector calculations with the experimental data of Wolfenstein parameters [20] listed in (12) from Particle Data Group [21], together with their relations with the four parameters, i.e., three mixing angles and one *CP*-violating phase.

$$\begin{aligned} &\sin\theta_{12} = \lambda, \\ &\sin\theta_{23} = A\lambda^2, \\ &\sin\theta_{13}e^{i\delta} = \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1-A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar\rho + i\bar\eta)]}, \\ &\lambda = 0.22535 \pm 0.00065, \\ &A = 0.811^{+0.022}_{-0.012}, \\ &\bar\rho = 0.131^{+0.026}_{-0.013}, \\ &\bar\eta = 0.345^{+0.013}_{-0.014}. \end{aligned} \tag{12}$$

From (12), we easily get mixing angles and *CP*-violating phase in P1:

$$P1: \quad \theta_{12} = \left(13.023^{+0.038}_{-0.038}\right)^\circ, \tag{13}$$
$$\theta_{23} = \left(2.360^{+0.065}_{-0.038}\right)^\circ, \tag{14}$$
$$\theta_{13} = \left(0.201^{+0.010}_{-0.008}\right)^\circ, \tag{15}$$
$$\phi_1 = \left(69.10^{+2.02}_{-3.85}\right)^\circ. \tag{16}$$

TABLE I: Nine different schemes of fermion mixing matrix

Scheme	Mixing angles and Jarlskog invariant
$\underline{P1: U = R_{23}(\theta_{23})R_{31}(\theta_{13},\phi)R_{12}(\theta_{12})}$ $\begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -c_{12}s_{23}s_{13}-s_{12}c_{23}e^{-i\phi} & -s_{12}s_{23}s_{13}+c_{12}c_{23}e^{-i\phi} & s_{23}c_{13} \\ -c_{12}s_{23}s_{13}+s_{12}s_{23}e^{-i\phi} & -s_{12}c_{23}s_{13}-c_{12}s_{23}e^{-i\phi} & c_{23}c_{13} \end{pmatrix}$	$J_1 = s_{12}s_{23}s_{13}c_{12}c_{23}c_{13}^2\sin\phi$ $\theta_{12} = \arcsin\frac{\lvert U_{12}\rvert}{\lvert U_{13}\rvert}$ $\theta_{23} = \arctan\frac{\lvert U_{23}\rvert}{\lvert U_{33}\rvert}$ $\theta_{13} = \arcsin\lvert U_{13}\rvert$
$\underline{P2: U = R_{12}(\theta_3)R_{23}(\theta_2,\phi)R_{12}^{-1}(\theta_1)}$ $\begin{pmatrix} s_1c_2s_3+c_1c_3e^{-i\phi} & c_1c_2s_3-s_1c_3e^{-i\phi} & s_2s_3 \\ s_1c_2c_3-c_1s_3e^{-i\phi} & c_1c_2c_3+s_1s_3e^{-i\phi} & s_2c_3 \\ -s_1s_2 & -c_1s_2 & c_2 \end{pmatrix}$	$J_2 = s_1s_2^2s_3c_1c_2c_3\sin\phi$ $\theta_1 = \arctan\frac{\lvert U_{31}\rvert}{\lvert U_{32}\rvert}$ $\theta_2 = \arccos\lvert U_{33}\rvert$ $\theta_3 = \arctan\frac{\lvert U_{13}\rvert}{\lvert U_{23}\rvert}$
$\underline{P3: U = R_{23}(\theta_2)R_{12}(\theta_1,\phi)R_{23}^{-1}(\theta_3)}$ $\begin{pmatrix} c_1 & s_1c_3 & -s_1s_3 \\ -s_1c_2 & c_1c_2c_3+s_2s_3e^{-i\phi} & -c_1c_2s_3+s_2c_3e^{-i\phi} \\ s_1s_2 & -c_1s_2c_3+c_2s_3e^{-i\phi} & c_1s_2s_3+c_2c_3e^{-i\phi} \end{pmatrix}$	$J_3 = s_1^2s_2s_3c_1c_2c_3\sin\phi$ $\theta_1 = \arccos\lvert U_{11}\rvert$ $\theta_2 = \arctan\frac{\lvert U_{31}\rvert}{\lvert U_{21}\rvert}$ $\theta_3 = \arctan\frac{\lvert U_{13}\rvert}{\lvert U_{12}\rvert}$
$\underline{P4: U = R_{23}(\theta_2)R_{12}(\theta_1,\phi)R_{31}^{-1}(\theta_3)}$ $\begin{pmatrix} c_1c_3 & s_1 & -c_1s_3 \\ -s_1c_2c_3+s_2s_3e^{-i\phi} & c_1c_2 & s_1c_2s_3+s_2c_3e^{-i\phi} \\ s_1s_2c_3+c_2s_3e^{-i\phi} & -c_1s_2 & -s_1s_2s_3+c_2c_3e^{-i\phi} \end{pmatrix}$	$J_4 = s_1s_2s_3c_1^2c_2c_3\sin\phi$ $\theta_1 = \arcsin\lvert U_{12}\rvert$ $\theta_2 = \arctan\frac{\lvert U_{32}\rvert}{\lvert U_{22}\rvert}$ $\theta_3 = \arctan\frac{\lvert U_{13}\rvert}{\lvert U_{11}\rvert}$
$\underline{P5: U = R_{31}(\theta_3)R_{23}(\theta_2,\phi)R_{12}^{-1}(\theta_1)}$ $\begin{pmatrix} -s_1s_2s_3+c_1c_3e^{-i\phi} & -c_1s_2s_3-s_1c_3e^{-i\phi} & c_2s_3 \\ s_1c_2 & c_1c_2 & s_2 \\ -s_1s_2c_3-c_1s_3e^{-i\phi} & -c_1s_2c_3+s_1s_3e^{-i\phi} & c_2c_3 \end{pmatrix}$	$J_5 = s_1s_2s_3c_1c_2^2c_3\sin\phi$ $\theta_1 = \arctan\frac{\lvert U_{21}\rvert}{\lvert U_{22}\rvert}$ $\theta_2 = \arcsin\lvert U_{23}\rvert$ $\theta_3 = \arctan\frac{\lvert U_{13}\rvert}{\lvert U_{33}\rvert}$
$\underline{P6: U = R_{12}(\theta_1)R_{31}(\theta_3,\phi)R_{23}^{-1}(\theta_2)}$ $\begin{pmatrix} c_1c_3 & c_1s_2s_3+s_1c_2e^{-i\phi} & c_1c_2s_3-s_1s_2e^{-i\phi} \\ -s_1c_3 & -s_1s_2s_3+c_1c_2e^{-i\phi} & -s_1c_2s_3-c_1s_2e^{-i\phi} \\ -s_3 & s_2c_3 & c_2c_3 \end{pmatrix}$	$J_6 = s_1s_2s_3c_1c_2c_3^2\sin\phi$ $\theta_1 = \arctan\frac{\lvert U_{21}\rvert}{\lvert U_{11}\rvert}$ $\theta_2 = \arctan\frac{\lvert U_{32}\rvert}{\lvert U_{33}\rvert}$ $\theta_3 = \arcsin\lvert U_{31}\rvert$
$\underline{P7: U = R_{31}(\theta_3)R_{12}(\theta_1,\phi)R_{31}^{-1}(\theta_2)}$ $\begin{pmatrix} c_1c_2c_3+s_2s_3e^{-i\phi} & s_1c_3 & -c_1s_2c_3+c_2s_3e^{-i\phi} \\ -s_1c_2 & c_1 & s_1s_2 \\ -c_1c_2s_3+s_2c_3e^{-i\phi} & -s_1s_3 & c_1s_2s_3+c_2c_3e^{-i\phi} \end{pmatrix}$	$J_7 = s_1^2s_2s_3c_1c_2c_3\sin\phi$ $\theta_1 = \arccos\lvert U_{22}\rvert$ $\theta_2 = \arctan\frac{\lvert U_{23}\rvert}{\lvert U_{21}\rvert}$ $\theta_3 = \arctan\frac{\lvert U_{32}\rvert}{\lvert U_{12}\rvert}$
$\underline{P8: U = R_{12}(\theta_1)R_{23}(\theta_2,\phi)R_{31}^{-1}(\theta_3)}$ $\begin{pmatrix} -s_1s_2s_3+c_1c_3e^{-i\phi} & s_1c_2 & s_1s_2c_3+c_1s_3e^{-i\phi} \\ -c_1s_2s_3-s_1c_3e^{-i\phi} & c_1c_2 & c_1s_2c_3-s_1s_3e^{-i\phi} \\ -c_2s_3 & -s_2 & c_2c_3 \end{pmatrix}$	$J_8 = s_1s_2s_3c_1c_2^2c_3\sin\phi$ $\theta_1 = \arctan\frac{\lvert U_{12}\rvert}{\lvert U_{22}\rvert}$ $\theta_2 = \arccos\lvert U_{32}\rvert$ $\theta_3 = \arctan\frac{\lvert U_{31}\rvert}{\lvert U_{33}\rvert}$
$\underline{P9: U = R_{31}(\theta_3)R_{12}(\theta_1,\phi)R_{23}^{-1}(\theta_2)}$ $\begin{pmatrix} c_1c_3 & s_1c_2c_3-s_2s_3e^{-i\phi} & s_1s_2c_3+c_2s_3e^{-i\phi} \\ -s_1 & c_1c_2 & c_1s_2 \\ -c_1s_3 & -s_1c_2s_3-s_2c_3e^{-i\phi} & -s_1s_2s_3+c_2c_3e^{-i\phi} \end{pmatrix}$	$J_9 = s_1s_2s_3c_1^2c_2c_3\sin\phi$ $\theta_1 = \arcsin\lvert U_{21}\rvert$ $\theta_2 = \arctan\frac{\lvert U_{23}\rvert}{\lvert U_{22}\rvert}$ $\theta_3 = \arctan\frac{\lvert U_{31}\rvert}{\lvert U_{11}\rvert}$

Next, the CKM matrix is calculated from the four parameters

$$V_{\mathrm{CKM}} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22535 \pm 0.00065 & 0.00352^{+0.00018}_{-0.00015} \\ 0.2252 \pm 0.0006 & 0.97344^{+0.00015}_{-0.00016} & 0.0412^{+0.0011}_{-0.0007} \\ 0.00867^{+0.00027}_{-0.00027} & 0.0404^{+0.0011}_{-0.0006} & 0.999145^{+0.000027}_{-0.000047} \end{pmatrix}. \tag{17}$$

When referred to matrix elements of the CKM matrix or the PMNS matrix, we always mean the absolute value of each matrix element in this article.

The Jarlskog invariant [22] is derived:

$$J = \sin\theta_{12}\sin\theta_{23}\sin\theta_{13}\cos\theta_{12}\cos\theta_{23}\cos^2\theta_{13}\sin\phi$$

$$= (2.97^{+0.17}_{-0.18}) \times 10^{-5}. \quad (18)$$

B. Lepton sector

The lepton sector is dealt with similarly, with the following normal hierarchy (NH) global fit data of mixing angles with 1σ errors in the P1 scheme [23]:

$$\begin{aligned} \sin^2\vartheta_{12} &= 0.307^{+0.018}_{-0.016}, \\ \sin^2\vartheta_{23} &= 0.386^{+0.024}_{-0.021}, \\ \sin^2\vartheta_{13} &= 0.0241^{+0.0025}_{-0.0025}, \end{aligned} \quad (19)$$

which are equivalently

$$\begin{aligned} \vartheta_{12} &= \left(33.65^{+1.11}_{-1.00}\right)^\circ, \\ \vartheta_{23} &= \left(38.41^{+1.40}_{-1.24}\right)^\circ, \\ \vartheta_{13} &= \left(8.93^{+0.46}_{-0.48}\right)^\circ. \end{aligned} \quad (20)$$

The inverse hierarchy global fit data of the P1 scheme mixing angles with 1σ errors [23] are

$$\begin{aligned} \sin^2\vartheta_{12} &= 0.307^{+0.018}_{-0.018}, \\ \sin^2\vartheta_{23} &= 0.392^{+0.039}_{-0.022}, \\ \sin^2\vartheta_{13} &= 0.0244^{+0.0023}_{-0.0025}. \end{aligned} \quad (21)$$

In this article, we only deal with the case of NH, because the global fit values for inverse hierarchy in (21) are quite close to the values for NH in (19) and thus our choice does not affect the analysis and conclusions of this article.

The difference from the quark sector is that at present there are no experimental results on the lepton *CP*-violating phase, but the four parameters are in a combined transformation when changing schemes. Therefore, to examine QLC and SC relations in the other eight schemes, it is necessary to choose a value of the lepton *CP*-violating phase. In this article we will not calculate the PMNS matrix under certain fixed value of the lepton *CP*-violating phase, such as the one with $\phi_3 = 90^\circ$ [24]. Instead, we will carry out the calculations with the *CP*-violating phase in the P3 scheme ϕ_3 varying almost continuously from 0° to 180°. (From now on we use ϕ_i with a subscript i to denote ϕ in the Pi scheme.) The results will be provided in tables with $\phi_3 = 0^\circ$, 45°, 90°, 135°, 180°, respectively, and in figures by smooth curves. To explicitly show our methods of calculation, we then carry out the calculations in detail in the case of the phase $\phi_3 = 90^\circ$.

First, we calculate the absolute values of five elements of the PMNS matrix that are independent of the lepton *CP*-violating phase from (19):

$$|U_{11}| = 0.822^{+0.010}_{-0.011}, \quad (22)$$
$$|U_{12}| = 0.547^{+0.016}_{-0.015}, \quad (23)$$
$$|U_{13}| = 0.155 \pm 0.008, \quad (24)$$
$$|U_{23}| = 0.614^{+0.019}_{-0.017}, \quad (25)$$
$$|U_{33}| = 0.774^{+0.013}_{-0.015}. \quad (26)$$

Then, the condition $\phi_3 = 90^\circ$ is used to determine mixing angles in the P3 scheme:

$$\begin{aligned} \cos\vartheta_1 &= |U_{11}| \\ \to \vartheta_1 &= \left(34.68^{+1.07}_{-0.97}\right)^\circ, \quad (27) \\ \tan\vartheta_3 &= \frac{|U_{13}|}{|U_{12}|} \\ \to \vartheta_3 &= \left(15.83^{+0.88}_{-0.93}\right)^\circ, \quad (28) \\ |U_{23}|^2 &= \frac{c_1^2 s_3^2 + c_3^2}{2} + \frac{c_1^2 s_3^2 - c_3^2}{2} \times \\ &\quad \cos(2\vartheta_2 + \arctan\frac{2c_1 s_3 c_3 \cos\phi_3}{c_1^2 s_3^2 - c_3^2}) \\ \to \vartheta_2 &= \left(37.64^{+1.58}_{-1.40}\right)^\circ. \quad (29) \end{aligned}$$

Next, we get all the elements of the PMNS matrix:

$$U_{\rm PMNS} = \begin{pmatrix} 0.822^{+0.010}_{-0.011} & 0.547^{+0.016}_{-0.015} & 0.155 \pm 0.008 \\ 0.451^{+0.014}_{-0.014} & 0.648^{+0.012}_{-0.014} & 0.614^{+0.019}_{-0.017} \\ 0.347^{+0.016}_{-0.014} & 0.529^{+0.015}_{-0.014} & 0.774^{+0.013}_{-0.015} \end{pmatrix}, \quad (30)$$

together with the Jarlskog invariant

$$J = 0.0338^{+0.0017}_{-0.0018}. \quad (31)$$

From the CKM and the PMNS matrices determined above, we then use the formulas in Table I to determine all the four parameters, i.e., three mixing angles and one *CP*-violating phase, in the other schemes in both the quark and the lepton sectors. Then we carry out calculations with other *CP*-violating phases by the same procedure. The results are listed in Table II. To better illustrate the dependence of the lepton mixing angles and *CP*-violating phases on ϕ_3, we also draw a series of graphs in Fig. 1. From Fig. 1, we can easily see that ϑ_1 in P2, ϑ_2 in P3, ϑ_2 in P4, ϑ_1 in P5, and all the mixing angles in P6$\sim$P9 have different values when different *CP*-violating phases in the KM scheme are adopted.

It is necessary to explain here the assumptions used in our calculations. Since generally there is just a slight dependence of the results of lepton mixing angles in the CK scheme on the lepton *CP*-violating phases, we simply assume that these results (19) are independent of the lepton *CP*-violating phases. Actually, for some experimental groups, their results and error bars of mixing angles in the CK scheme actually vary with different phase assumptions. (See for example Ref. [25] and Ref. [26].) This proves that the independence is suitable only approximately.

III. QUARK-LEPTON COMPLEMENTARITY

With the quark and lepton mixing angles calculated in the previous section, we now go on to discuss the topic of QLC. A diagrammatic presentation of QLC is shown in Fig. 2. We list the results in Table III with five different

TABLE II: Mixing angles and *CP*-violating phases in different schemes

	quark	lepton($\phi_3 = 0^\circ$)	lepton($\phi_3 = 45^\circ$)	lepton($\phi_3 = 90^\circ$)	lepton($\phi_3 = 135^\circ$)	lepton($\phi_3 = 180^\circ$)
P1	$\theta_{12} = (13.023^{+0.038}_{-0.038})^\circ$	$\vartheta_{12} = (33.65^{+1.11}_{-1.00})^\circ$	$\vartheta_{12} = (33.65^{+1.11}_{-1.00})^\circ$	$\vartheta_{12} = (33.65^{+1.11}_{-1.00})^\circ$	$\vartheta_{12} = (33.65^{+1.11}_{-1.00})^\circ$	$\vartheta_{12} = (33.65^{+1.11}_{-1.00})^\circ$
	$\theta_{23} = (2.360^{+0.065}_{-0.038})^\circ$	$\vartheta_{23} = (38.41^{+1.40}_{-1.24})^\circ$	$\vartheta_{23} = (38.41^{+1.40}_{-1.24})^\circ$	$\vartheta_{23} = (38.41^{+1.40}_{-1.24})^\circ$	$\vartheta_{23} = (38.41^{+1.40}_{-1.24})^\circ$	$\vartheta_{23} = (38.41^{+1.40}_{-1.24})^\circ$
	$\theta_{13} = (0.201^{+0.010}_{-0.008})^\circ$	$\vartheta_{13} = (8.93^{+0.46}_{-0.48})^\circ$	$\vartheta_{13} = (8.93^{+0.46}_{-0.48})^\circ$	$\vartheta_{13} = (8.93^{+0.46}_{-0.48})^\circ$	$\vartheta_{13} = (8.93^{+0.46}_{-0.48})^\circ$	$\vartheta_{13} = (8.93^{+0.46}_{-0.48})^\circ$
	$\phi_1 = (69.10^{+2.02}_{-3.85})^\circ$	$\phi_1 = 180^\circ$	$\phi_1 = (133.70^{+0.98}_{-0.88})^\circ$	$\phi_1 = (83.37^{+4.96}_{-1.40})^\circ$	$\phi_1 = (37.46^{+1.20}_{-1.11})^\circ$	$\phi_1 = 0^\circ$
P2	$\theta_1 = (12.109^{+0.174}_{-0.327})^\circ$	$\vartheta_1 = (44.73^{+1.33}_{-1.27})^\circ$	$\vartheta_1 = (41.64^{+1.16}_{-1.08})^\circ$	$\vartheta_1 = (33.28^{+1.10}_{-1.00})^\circ$	$\vartheta_1 = (25.34^{+1.31}_{-1.20})^\circ$	$\vartheta_1 = (22.57^{+1.36}_{-1.24})^\circ$
	$\theta_2 = (2.369^{+0.065}_{-0.037})^\circ$	$\vartheta_2 = (39.28^{+1.37}_{-1.21})^\circ$	$\vartheta_2 = (39.28^{+1.37}_{-1.21})^\circ$	$\vartheta_2 = (39.28^{+1.37}_{-1.21})^\circ$	$\vartheta_2 = (39.28^{+1.37}_{-1.21})^\circ$	$\vartheta_2 = (39.28^{+1.37}_{-1.21})^\circ$
	$\theta_3 = (4.880^{+0.256}_{-0.277})^\circ$	$\vartheta_3 = (14.19^{+0.80}_{-0.85})^\circ$	$\vartheta_3 = (14.19^{+0.80}_{-0.85})^\circ$	$\vartheta_3 = (14.19^{+0.80}_{-0.85})^\circ$	$\vartheta_3 = (14.19^{+0.80}_{-0.85})^\circ$	$\vartheta_3 = (14.19^{+0.80}_{-0.85})^\circ$
	$\phi_2 = (89.69^{+2.29}_{-3.95})^\circ$	$\phi_2 = 0^\circ$	$\phi_2 = (42.20^{+0.99}_{-1.01})^\circ$	$\phi_2 = (87.07^{+1.05}_{-1.10})^\circ$	$\phi_2 = (133.51^{+0.54}_{-0.57})^\circ$	$\phi_2 = 180^\circ$
P3	$\theta_1 = (13.025^{+0.038}_{-0.038})^\circ$	$\vartheta_1 = (34.68^{+1.07}_{-0.97})^\circ$	$\vartheta_1 = (34.68^{+1.07}_{-0.97})^\circ$	$\vartheta_1 = (34.68^{+1.07}_{-0.97})^\circ$	$\vartheta_1 = (34.68^{+1.07}_{-0.97})^\circ$	$\vartheta_1 = (34.68^{+1.07}_{-0.97})^\circ$
	$\theta_2 = (2.205^{+0.068}_{-0.068})^\circ$	$\vartheta_2 = (51.54^{+1.62}_{-1.51})^\circ$	$\vartheta_2 = (47.67^{+1.60}_{-1.45})^\circ$	$\vartheta_2 = (37.64^{+1.58}_{-1.40})^\circ$	$\vartheta_2 = (28.44^{+1.64}_{-1.48})^\circ$	$\vartheta_2 = (25.28^{+1.64}_{-1.48})^\circ$
	$\theta_3 = (0.894^{+0.045}_{-0.045})^\circ$	$\vartheta_3 = (15.83^{+0.88}_{-0.93})^\circ$	$\vartheta_3 = (15.83^{+0.88}_{-0.93})^\circ$	$\vartheta_3 = (15.83^{+0.88}_{-0.93})^\circ$	$\vartheta_3 = (15.83^{+0.88}_{-0.93})^\circ$	$\vartheta_3 = (15.83^{+0.88}_{-0.93})^\circ$
	$\phi_3 = (89.29^{+3.99}_{-2.33})^\circ$	$\phi_3 = 0^\circ$	$\phi_3 = 45.00^\circ$	$\phi_3 = 90.00^\circ$	$\phi_3 = 135.00^\circ$	$\phi_3 = 180^\circ$
P4	$\theta_1 = (13.023^{+0.038}_{-0.038})^\circ$	$\vartheta_1 = (33.19^{+1.09}_{-0.99})^\circ$	$\vartheta_1 = (33.19^{+1.09}_{-0.99})^\circ$	$\vartheta_1 = (33.19^{+1.09}_{-0.99})^\circ$	$\vartheta_1 = (33.19^{+1.09}_{-0.99})^\circ$	$\vartheta_1 = (33.19^{+1.09}_{-0.99})^\circ$
	$\theta_2 = (2.377^{+0.066}_{-0.038})^\circ$	$\vartheta_2 = (32.51^{+1.46}_{-1.30})^\circ$	$\vartheta_2 = (34.43^{+1.34}_{-1.19})^\circ$	$\vartheta_2 = (39.23^{+1.23}_{-1.09})^\circ$	$\vartheta_2 = (43.13^{+1.37}_{-1.22})^\circ$	$\vartheta_2 = (44.31^{+1.46}_{-1.30})^\circ$
	$\theta_3 = (0.207^{+0.010}_{-0.010})^\circ$	$\vartheta_3 = (10.69^{+0.56}_{-0.58})^\circ$	$\vartheta_3 = (10.69^{+0.56}_{-0.58})^\circ$	$\vartheta_3 = (10.69^{+0.56}_{-0.58})^\circ$	$\vartheta_3 = (10.69^{+0.56}_{-0.58})^\circ$	$\vartheta_3 = (10.69^{+0.56}_{-0.58})^\circ$
	$\phi_4 = (111.95^{+3.82}_{-2.02})^\circ$	$\phi_4 = 0^\circ$	$\phi_4 = (49.01^{+1.36}_{-1.48})^\circ$	$\phi_4 = (99.20^{+1.88}_{-2.08})^\circ$	$\phi_4 = (143.60^{+1.33}_{-1.47})^\circ$	$\phi_4 = 180^\circ$
P5	$\theta_1 = (13.026^{+0.038}_{-0.038})^\circ$	$\vartheta_1 = (26.63^{+1.21}_{-1.12})^\circ$	$\vartheta_1 = (29.03^{+1.15}_{-1.06})^\circ$	$\vartheta_1 = (34.80^{+1.05}_{-0.95})^\circ$	$\vartheta_1 = (39.32^{+1.14}_{-1.04})^\circ$	$\vartheta_1 = (40.66^{+1.22}_{-1.11})^\circ$
	$\theta_2 = (2.360^{+0.065}_{-0.038})^\circ$	$\vartheta_2 = (37.86^{+1.38}_{-1.22})^\circ$	$\vartheta_2 = (37.86^{+1.38}_{-1.22})^\circ$	$\vartheta_2 = (37.86^{+1.38}_{-1.22})^\circ$	$\vartheta_2 = (37.86^{+1.38}_{-1.22})^\circ$	$\vartheta_2 = (37.86^{+1.38}_{-1.22})^\circ$
	$\theta_3 = (0.202^{+0.010}_{-0.010})^\circ$	$\vartheta_3 = (11.34^{+0.61}_{-0.63})^\circ$	$\vartheta_3 = (11.34^{+0.61}_{-0.63})^\circ$	$\vartheta_3 = (11.34^{+0.61}_{-0.63})^\circ$	$\vartheta_3 = (11.34^{+0.61}_{-0.63})^\circ$	$\vartheta_3 = (11.34^{+0.61}_{-0.63})^\circ$
	$\phi_5 = (110.94^{+3.85}_{-2.02})^\circ$	$\phi_5 = 0^\circ$	$\phi_5 = (51.82^{+0.80}_{-0.86})^\circ$	$\phi_5 = (102.03^{+1.46}_{-1.57})^\circ$	$\phi_5 = (145.09^{+1.17}_{-1.26})^\circ$	$\phi_5 = 180^\circ$
P6	$\theta_1 = (13.016^{+0.038}_{-0.038})^\circ$	$\vartheta_1 = (23.28^{+1.23}_{-1.18})^\circ$	$\vartheta_1 = (24.98^{+1.08}_{-1.13})^\circ$	$\vartheta_1 = (28.72^{+1.05}_{-1.00})^\circ$	$\vartheta_1 = (31.31^{+0.99}_{-0.92})^\circ$	$\vartheta_1 = (32.03^{+0.98}_{-0.91})^\circ$
	$\theta_2 = (2.316^{+0.064}_{-0.037})^\circ$	$\vartheta_2 = (30.16^{+1.52}_{-1.37})^\circ$	$\vartheta_2 = (31.44^{+1.41}_{-1.26})^\circ$	$\vartheta_2 = (34.36^{+1.27}_{-1.13})^\circ$	$\vartheta_2 = (36.47^{+1.25}_{-1.12})^\circ$	$\vartheta_2 = (37.06^{+1.26}_{-1.13})^\circ$
	$\theta_3 = (0.497^{+0.015}_{-0.015})^\circ$	$\vartheta_3 = (26.46^{+0.85}_{-0.79})^\circ$	$\vartheta_3 = (24.87^{+0.87}_{-0.80})^\circ$	$\vartheta_3 = (20.33^{+0.96}_{-0.87})^\circ$	$\vartheta_3 = (15.72^{+1.04}_{-0.94})^\circ$	$\vartheta_3 = (14.06^{+1.05}_{-0.95})^\circ$
	$\phi_6 = (22.72^{+1.25}_{-1.18})^\circ$	$\phi_6 = 0^\circ$	$\phi_6 = (24.65^{+1.40}_{-1.47})^\circ$	$\phi_6 = (34.29^{+1.92}_{-2.02})^\circ$	$\phi_6 = (22.85^{+1.16}_{-1.23})^\circ$	$\phi_6 = 0^\circ$
P7	$\theta_1 = (13.235^{+0.039}_{-0.038})^\circ$	$\vartheta_1 = (45.11^{+1.09}_{-0.95})^\circ$	$\vartheta_1 = (46.35^{+1.06}_{-0.93})^\circ$	$\vartheta_1 = (49.59^{+1.06}_{-0.93})^\circ$	$\vartheta_1 = (52.36^{+1.20}_{-1.06})^\circ$	$\vartheta_1 = (53.21^{+1.27}_{-1.12})^\circ$
	$\theta_2 = (10.363^{+0.283}_{-0.164})^\circ$	$\vartheta_2 = (60.03^{+1.76}_{-1.69})^\circ$	$\vartheta_2 = (58.03^{+1.71}_{-1.63})^\circ$	$\vartheta_2 = (53.72^{+1.56}_{-1.46})^\circ$	$\vartheta_2 = (50.82^{+1.40}_{-1.31})^\circ$	$\vartheta_2 = (50.03^{+1.35}_{-1.27})^\circ$
	$\theta_3 = (10.167^{+0.276}_{-0.160})^\circ$	$\vartheta_3 = (39.41^{+1.64}_{-1.67})^\circ$	$\vartheta_3 = (40.84^{+1.50}_{-1.54})^\circ$	$\vartheta_3 = (44.04^{+1.31}_{-1.36})^\circ$	$\vartheta_3 = (46.27^{+1.23}_{-1.28})^\circ$	$\vartheta_3 = (46.88^{+1.21}_{-1.25})^\circ$
	$\phi_7 = (1.08^{+0.06}_{-0.06})^\circ$	$\phi_7 = 0^\circ$	$\phi_7 = (17.84^{+0.96}_{-1.00})^\circ$	$\phi_7 = (22.16^{+1.18}_{-1.20})^\circ$	$\phi_7 = (12.76^{+0.71}_{-0.68})^\circ$	$\phi_7 = 0^\circ$
P8	$\theta_1 = (13.034^{+0.038}_{-0.038})^\circ$	$\vartheta_1 = (37.80^{+1.17}_{-1.05})^\circ$	$\vartheta_1 = (38.41^{+1.20}_{-1.08})^\circ$	$\vartheta_1 = (40.17^{+1.27}_{-1.15})^\circ$	$\vartheta_1 = (41.87^{+1.41}_{-1.27})^\circ$	$\vartheta_1 = (42.43^{+1.48}_{-1.32})^\circ$
	$\theta_2 = (2.316^{+0.064}_{-0.037})^\circ$	$\vartheta_2 = (26.73^{+1.23}_{-1.15})^\circ$	$\vartheta_2 = (28.24^{+1.13}_{-1.05})^\circ$	$\vartheta_2 = (31.95^{+1.02}_{-0.94})^\circ$	$\vartheta_2 = (34.90^{+1.07}_{-0.99})^\circ$	$\vartheta_2 = (35.77^{+1.10}_{-1.01})^\circ$
	$\theta_3 = (0.497^{+0.015}_{-0.015})^\circ$	$\vartheta_3 = (29.92^{+1.13}_{-1.01})^\circ$	$\vartheta_3 = (28.52^{+1.19}_{-1.05})^\circ$	$\vartheta_3 = (24.17^{+1.33}_{-1.18})^\circ$	$\vartheta_3 = (19.29^{+1.44}_{-1.27})^\circ$	$\vartheta_3 = (17.43^{+1.45}_{-1.28})^\circ$
	$\phi_8 = (157.31^{+1.18}_{-1.25})^\circ$	$\phi_8 = 180^\circ$	$\phi_8 = (160.86^{+1.27}_{-1.20})^\circ$	$\phi_8 = (151.21^{+1.79}_{-1.70})^\circ$	$\phi_8 = (159.70^{+1.14}_{-1.08})^\circ$	$\phi_8 = 180^\circ$
P9	$\theta_1 = (13.015^{+0.038}_{-0.038})^\circ$	$\vartheta_1 = (20.72^{+1.05}_{-1.04})^\circ$	$\vartheta_1 = (22.53^{+1.02}_{-1.02})^\circ$	$\vartheta_1 = (26.78^{+0.93}_{-0.93})^\circ$	$\vartheta_1 = (30.02^{+0.89}_{-0.87})^\circ$	$\vartheta_1 = (30.96^{+0.89}_{-0.86})^\circ$
	$\theta_2 = (2.423^{+0.067}_{-0.038})^\circ$	$\vartheta_2 = (41.01^{+1.35}_{-1.21})^\circ$	$\vartheta_2 = (41.64^{+1.36}_{-1.21})^\circ$	$\vartheta_2 = (43.43^{+1.27}_{-1.12})^\circ$	$\vartheta_2 = (45.14^{+1.57}_{-1.40})^\circ$	$\vartheta_2 = (45.70^{+1.64}_{-1.46})^\circ$
	$\theta_3 = (0.510^{+0.016}_{-0.016})^\circ$	$\vartheta_3 = (28.45^{+0.96}_{-0.88})^\circ$	$\vartheta_3 = (27.09^{+0.99}_{-0.90})^\circ$	$\vartheta_3 = (22.90^{+1.12}_{-1.01})^\circ$	$\vartheta_3 = (18.24^{+1.25}_{-1.13})^\circ$	$\vartheta_3 = (16.46^{+2.70}_{-1.38})^\circ$
	$\phi_9 = (158.32^{+1.13}_{-1.20})^\circ$	$\phi_9 = 180^\circ$	$\phi_9 = (158.05^{+1.24}_{-1.17})^\circ$	$\phi_9 = (148.28^{+1.87}_{-1.77})^\circ$	$\phi_9 = (158.21^{+1.27}_{-1.14})^\circ$	$\phi_9 = 180^\circ$

CP-violating phases, i.e., $\phi_3 = 0^\circ$, 45°, 90°, 135°, and 180°, respectively.

Here, we distinguish two types of the behavior of the sums by symbols A and B, respectively:

Type A The values are independent of the lepton *CP*-violating phase ϕ_3.

Type B The values vary with the variation of the lepton *CP*-violating phase ϕ_3.

The subscripts of each type represent the error limit. (In the case of Type B, we classify the deviations only by the values in the condition $\phi_3 = 90^\circ$.) For example, A_3 represents that the sum in Type A deviates from 45°

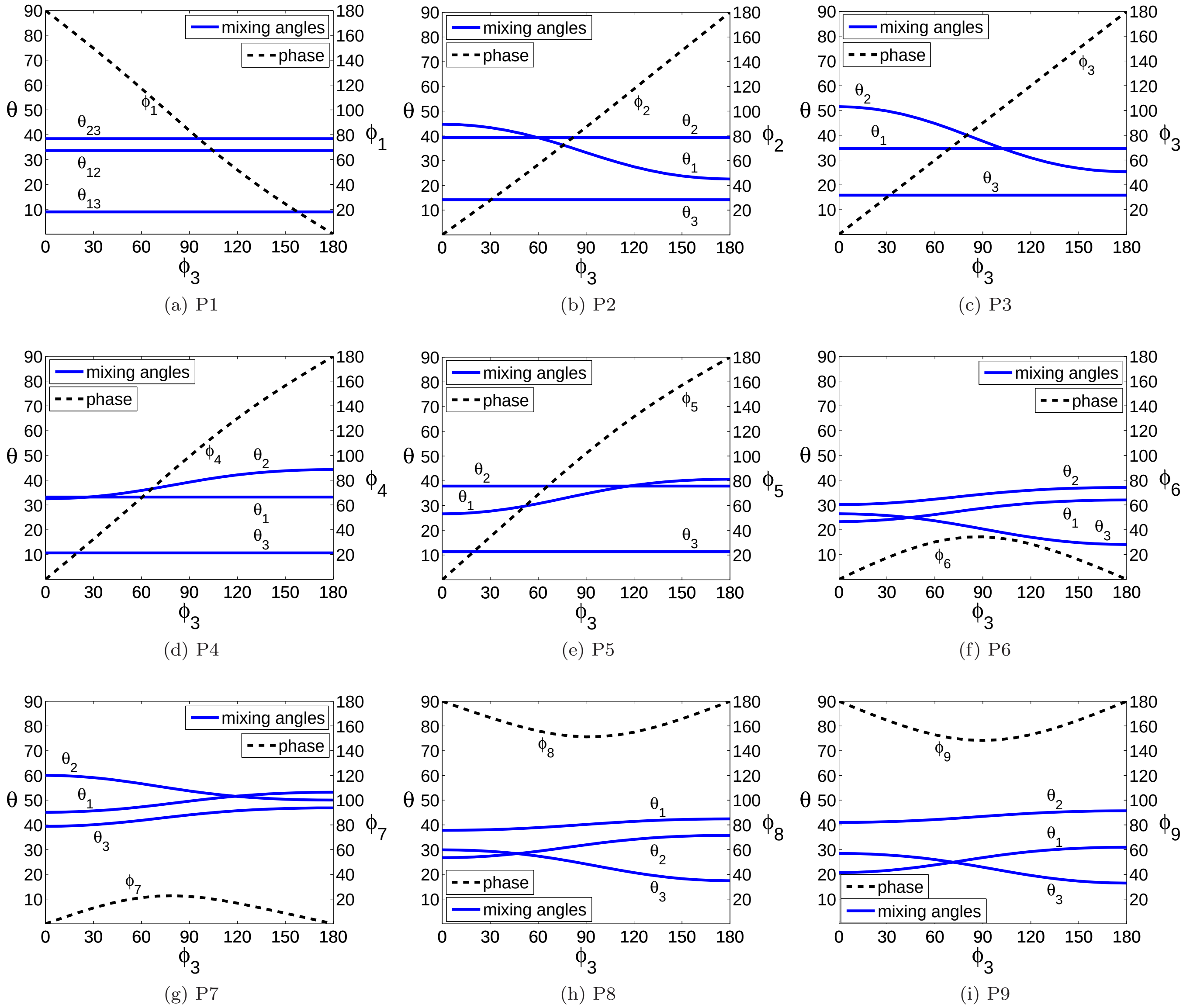
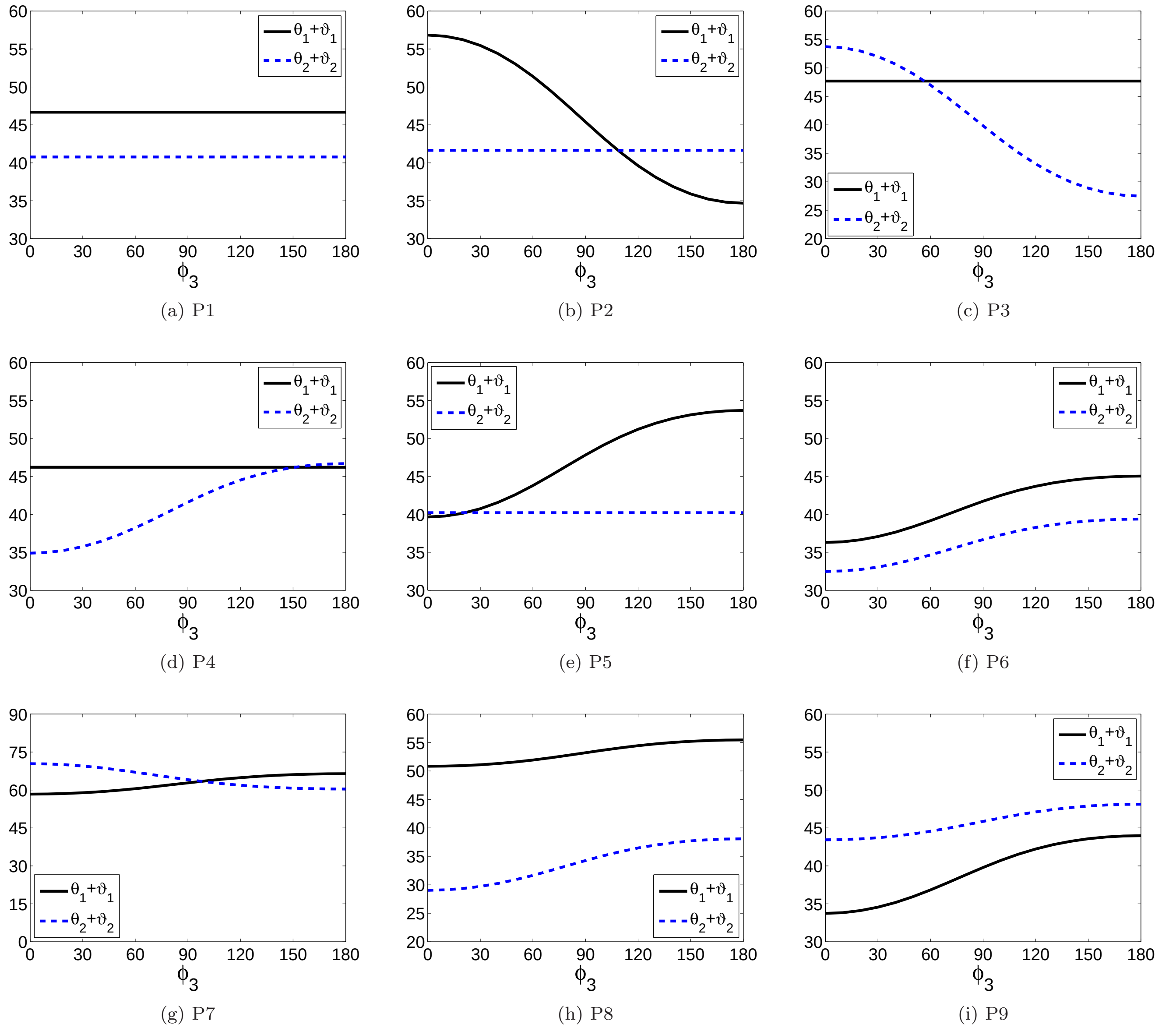


FIG. 1: Mixing angles and lepton CP-violating phases in different schemes (All the values are in the unit of degree (°).)

with an error between 2σ and 3σ; $B_{>5}$ means that the sum with $\phi_3 = 90°$ in Type B deviates from 45° with an error larger than 5σ.

All the relations in Type A are relatively more consistent with the prediction of QLC, while the phase-dependent property of Type B relations adds complexities. Moreover, many Type B relations in Table III largely deviate from expectations. We remind readers to pay special attention to the P1, P7, and P8 schemes. In P1, QLC1 are obviously in Type A and are close to the expected value 45°, which in fact is a major cause leading to the hypothesis of QLC. However, with the latest global fit data [23], QLC2 in P1 deviates from expectations with an error larger than 3σ and thus even in the P1 scheme, QLC2 may not be good relations, which is obscured by relatively less accurate data before. In P7 and P8, all QLC relations are far beyond error limits, no matter what value of ϕ_3 we choose, and thus are hardly desired relations. Therefore, we see that the validation of QLC in some schemes significantly depends on the lepton CP-violating phase ϕ_3 we choose, and in the P7 and the P8 schemes, QLC can never be satisfied. This dependence of QLC on the choices of schemes and lepton CP-violating phase was sometimes ignored by previous works.

Since the QLC relations are originally observed in the standard CK scheme, this phase-dependent property and the generally phase-dependent result of QLC in the other eight schemes remind us to be cautious on the generalization of QLC from the CK scheme to the other eight schemes. When considering such generalizations, careful inspections and justifications should be carried out. In



(a) P1 (b) P2 (c) P3 (d) P4 (e) P5 (f) P6 (g) P7 (h) P8 (i) P9

FIG. 2: The quark-lepton complementarity between mixing angles of quarks and leptons [All the values are in the unit of degree (°).]

addition, Jarlskog has pointed out a few of the uncertainties that could invalidate the QLC analyses [27], which also reminds us to carefully treat QLC relations. An alternative way of avoiding such generalizations is to use some scheme-independent forms of QLC relations. One example is to analyze QLC relations in the form of matrix elements [13, 14, 28]. Since we have figured out the dependence of QLC relations on the *CP*-violating phase, experimental results on the lepton *CP*-violating phase measured in the future will be helpful in analyzing QLC in the other eight schemes.

IV. SELF-COMPLEMENTARITY

SC relations of lepton mixing angles are examined similarly as QLC, with the results shown in Fig. 3. More detailed results with errors are provided in Table IV. The classification into two types and the definition of subscripts follow our treatment with QLC, and the values of ϑ_2 are included for reference. Here, we also remind readers to pay attention to P1, P7, and P8 results. The former one fits relatively well, while the latter two are hardly desired relations. Similarly, we should be cautious about the generalization of SC relations to the other eight schemes, and it is clear that experimental results of the lepton *CP*-violating phase will be helpful in the exami-

TABLE III: The quark-lepton complementarity between quark and lepton mixing angles

	QLC	$(\phi_3 = 0^\circ)$	$(\phi_3 = 45^\circ)$	$(\phi_3 = 90^\circ)$	$(\phi_3 = 135^\circ)$	$(\phi_3 = 180^\circ)$	Type
P1	$\theta_{12} + \vartheta_{12}$	$(46.65^{+1.11}_{-1.00})^\circ$	$(46.65^{+1.11}_{-1.00})^\circ$	$(46.65^{+1.11}_{-1.00})^\circ$	$(46.65^{+1.11}_{-1.00})^\circ$	$(46.65^{+1.11}_{-1.00})^\circ$	A_1
	$\theta_{23} + \vartheta_{23}$	$(40.77^{+1.40}_{-1.24})^\circ$	$(40.77^{+1.40}_{-1.24})^\circ$	$(40.77^{+1.40}_{-1.24})^\circ$	$(40.77^{+1.40}_{-1.24})^\circ$	$(40.77^{+1.40}_{-1.24})^\circ$	A_4
P2	$\theta_1 + \vartheta_1$	$(56.84^{+1.33}_{-1.27})^\circ$	$(53.75^{+1.16}_{-1.08})^\circ$	$(45.39^{+1.10}_{-1.00})^\circ$	$(37.45^{+1.31}_{-1.20})^\circ$	$(34.68^{+1.36}_{-1.24})^\circ$	B_1
	$\theta_2 + \vartheta_2$	$(41.65^{+1.33}_{-1.27})^\circ$	$(41.65^{+1.33}_{-1.27})^\circ$	$(41.65^{+1.33}_{-1.27})^\circ$	$(41.65^{+1.33}_{-1.27})^\circ$	$(41.65^{+1.33}_{-1.27})^\circ$	A_3
P3	$\theta_1 + \vartheta_1$	$(47.71^{+1.07}_{-0.97})^\circ$	$(47.71^{+1.07}_{-0.97})^\circ$	$(47.71^{+1.07}_{-0.97})^\circ$	$(47.71^{+1.07}_{-0.97})^\circ$	$(47.71^{+1.07}_{-0.97})^\circ$	A_3
	$\theta_2 + \vartheta_2$	$(53.75^{+1.62}_{-1.51})^\circ$	$(49.88^{+1.60}_{-1.45})^\circ$	$(39.85^{+1.58}_{-1.40})^\circ$	$(30.65^{+1.64}_{-1.48})^\circ$	$(27.49^{+1.64}_{-1.48})^\circ$	B_4
P4	$\theta_1 + \vartheta_1$	$(46.21^{+1.09}_{-0.99})^\circ$	$(46.21^{+1.09}_{-0.99})^\circ$	$(46.21^{+1.09}_{-0.99})^\circ$	$(46.21^{+1.09}_{-0.99})^\circ$	$(46.21^{+1.09}_{-0.99})^\circ$	A_2
	$\theta_2 + \vartheta_2$	$(34.89^{+1.46}_{-1.30})^\circ$	$(36.81^{+1.34}_{-1.19})^\circ$	$(41.61^{+1.23}_{-1.09})^\circ$	$(45.51^{+1.37}_{-1.22})^\circ$	$(46.69^{+1.46}_{-1.30})^\circ$	B_4
P5	$\theta_1 + \vartheta_1$	$(39.64^{+1.21}_{-1.12})^\circ$	$(42.06^{+1.15}_{-1.06})^\circ$	$(47.83^{+1.05}_{-0.95})^\circ$	$(52.35^{+1.14}_{-1.04})^\circ$	$(53.69^{+1.22}_{-1.11})^\circ$	B_3
	$\theta_2 + \vartheta_2$	$(40.22^{+1.38}_{-1.22})^\circ$	$(40.22^{+1.38}_{-1.22})^\circ$	$(40.22^{+1.38}_{-1.22})^\circ$	$(40.22^{+1.38}_{-1.22})^\circ$	$(40.22^{+1.38}_{-1.22})^\circ$	A_4
P6	$\theta_1 + \vartheta_1$	$(36.30^{+1.23}_{-1.18})^\circ$	$(38.00^{+1.08}_{-1.13})^\circ$	$(41.74^{+1.05}_{-1.00})^\circ$	$(44.33^{+0.99}_{-0.92})^\circ$	$(45.05^{+0.98}_{-0.91})^\circ$	B_4
	$\theta_2 + \vartheta_2$	$(32.48^{+1.52}_{-1.37})^\circ$	$(33.76^{+1.41}_{-1.26})^\circ$	$(36.68^{+1.27}_{-1.13})^\circ$	$(38.79^{+1.25}_{-1.12})^\circ$	$(39.38^{+1.26}_{-1.13})^\circ$	$B_{>5}$
P7	$\theta_1 + \vartheta_1$	$(58.35^{+1.09}_{-0.95})^\circ$	$(59.59^{+1.06}_{-0.93})^\circ$	$(62.83^{+1.06}_{-0.93})^\circ$	$(65.60^{+1.20}_{-1.06})^\circ$	$(66.45^{+1.27}_{-1.12})^\circ$	$B_{>5}$
	$\theta_2 + \vartheta_2$	$(70.39^{+1.76}_{-1.69})^\circ$	$(68.39^{+1.71}_{-1.63})^\circ$	$(64.08^{+1.56}_{-1.46})^\circ$	$(61.18^{+1.40}_{-1.31})^\circ$	$(60.39^{+1.35}_{-1.27})^\circ$	$B_{>5}$
P8	$\theta_1 + \vartheta_1$	$(50.83^{+1.17}_{-1.05})^\circ$	$(51.44^{+1.20}_{-1.08})^\circ$	$(53.20^{+1.27}_{-1.15})^\circ$	$(54.90^{+1.41}_{-1.27})^\circ$	$(55.46^{+1.48}_{-1.32})^\circ$	$B_{>5}$
	$\theta_2 + \vartheta_2$	$(29.05^{+1.23}_{-1.15})^\circ$	$(30.56^{+1.13}_{-1.05})^\circ$	$(34.27^{+1.02}_{-0.94})^\circ$	$(37.22^{+1.07}_{-0.99})^\circ$	$(38.09^{+1.10}_{-1.01})^\circ$	$B_{>5}$
P9	$\theta_1 + \vartheta_1$	$(33.74^{+1.05}_{-1.04})^\circ$	$(35.55^{+1.02}_{-1.02})^\circ$	$(39.80^{+0.93}_{-0.93})^\circ$	$(43.04^{+0.89}_{-0.87})^\circ$	$(43.98^{+0.89}_{-0.86})^\circ$	$B_{>5}$
	$\theta_2 + \vartheta_2$	$(43.43^{+1.35}_{-1.21})^\circ$	$(44.06^{+1.36}_{-1.21})^\circ$	$(45.85^{+1.27}_{-1.12})^\circ$	$(47.56^{+1.57}_{-1.40})^\circ$	$(48.12^{+1.64}_{-1.46})^\circ$	B_1

TABLE IV: The self-complementarity among lepton mixing angles

	SC	$(\phi_3 = 0^\circ)$	$(\phi_3 = 45^\circ)$	$(\phi_3 = 90^\circ)$	$(\phi_3 = 135^\circ)$	$(\phi_3 = 180^\circ)$	Type
P1	$\vartheta_{12} + \vartheta_{13}$	$(42.58^{+1.20}_{-1.11})^\circ$	$(42.58^{+1.20}_{-1.11})^\circ$	$(42.58^{+1.20}_{-1.11})^\circ$	$(42.58^{+1.20}_{-1.11})^\circ$	$(42.58^{+1.20}_{-1.11})^\circ$	A_3
	ϑ_{23}	$(38.41^{+1.40}_{-1.24})^\circ$	$(38.41^{+1.40}_{-1.24})^\circ$	$(38.41^{+1.40}_{-1.24})^\circ$	$(38.41^{+1.40}_{-1.24})^\circ$	$(38.41^{+1.40}_{-1.24})^\circ$	
P2	$\vartheta_1 + \vartheta_3$	$(58.92^{+1.56}_{-1.53})^\circ$	$(55.83^{+1.41}_{-1.38})^\circ$	$(47.48^{+1.36}_{-1.31})^\circ$	$(39.53^{+1.54}_{-1.47})^\circ$	$(36.76^{+1.58}_{-1.50})^\circ$	B_2
	ϑ_2	$(39.28^{+1.37}_{-1.21})^\circ$	$(39.28^{+1.37}_{-1.21})^\circ$	$(39.28^{+1.37}_{-1.21})^\circ$	$(39.28^{+1.37}_{-1.21})^\circ$	$(39.28^{+1.37}_{-1.21})^\circ$	
P3	$\vartheta_1 + \vartheta_3$	$(50.51^{+1.39}_{-1.34})^\circ$	$(50.51^{+1.39}_{-1.34})^\circ$	$(50.51^{+1.39}_{-1.34})^\circ$	$(50.51^{+1.39}_{-1.34})^\circ$	$(50.51^{+1.39}_{-1.34})^\circ$	A_5
	ϑ_2	$(51.54^{+1.62}_{-1.51})^\circ$	$(47.67^{+1.60}_{-1.45})^\circ$	$(37.64^{+1.58}_{-1.40})^\circ$	$(28.44^{+1.64}_{-1.48})^\circ$	$(25.28^{+1.64}_{-1.48})^\circ$	
P4	$\vartheta_1 + \vartheta_3$	$(43.88^{+1.23}_{-1.14})^\circ$	$(43.88^{+1.23}_{-1.14})^\circ$	$(43.88^{+1.23}_{-1.14})^\circ$	$(43.88^{+1.23}_{-1.14})^\circ$	$(43.88^{+1.23}_{-1.14})^\circ$	A_1
	ϑ_2	$(32.51^{+1.46}_{-1.30})^\circ$	$(34.43^{+1.34}_{-1.19})^\circ$	$(39.23^{+1.23}_{-1.09})^\circ$	$(43.13^{+1.37}_{-1.22})^\circ$	$(44.31^{+1.46}_{-1.30})^\circ$	
P5	$\vartheta_1 + \vartheta_3$	$(37.97^{+1.36}_{-1.28})^\circ$	$(40.37^{+1.30}_{-1.23})^\circ$	$(46.14^{+1.21}_{-1.14})^\circ$	$(50.66^{+1.29}_{-1.21})^\circ$	$(52.00^{+1.36}_{-1.28})^\circ$	B_1
	ϑ_2	$(37.86^{+1.38}_{-1.22})^\circ$	$(37.86^{+1.38}_{-1.22})^\circ$	$(37.86^{+1.38}_{-1.22})^\circ$	$(37.86^{+1.38}_{-1.22})^\circ$	$(37.86^{+1.38}_{-1.22})^\circ$	
P6	$\vartheta_1 + \vartheta_3$	$(49.74^{+1.50}_{-1.42})^\circ$	$(49.85^{+1.47}_{-1.39})^\circ$	$(49.05^{+1.42}_{-1.33})^\circ$	$(47.04^{+1.44}_{-1.32})^\circ$	$(46.09^{+1.43}_{-1.31})^\circ$	B_4
	ϑ_2	$(30.16^{+1.52}_{-1.37})^\circ$	$(31.44^{+1.41}_{-1.26})^\circ$	$(34.36^{+1.27}_{-1.13})^\circ$	$(36.47^{+1.25}_{-1.12})^\circ$	$(37.06^{+1.26}_{-1.13})^\circ$	
P7	$\vartheta_1 + \vartheta_3$	$(84.52^{+1.96}_{-1.92})^\circ$	$(87.19^{+1.84}_{-1.80})^\circ$	$(93.62^{+1.68}_{-1.64})^\circ$	$(98.63^{+1.72}_{-1.66})^\circ$	$(100.10^{+1.76}_{-1.68})^\circ$	$B_{>5}$
	ϑ_2	$(60.03^{+1.76}_{-1.69})^\circ$	$(58.03^{+1.71}_{-1.63})^\circ$	$(53.72^{+1.56}_{-1.46})^\circ$	$(50.82^{+1.40}_{-1.31})^\circ$	$(50.03^{+1.35}_{-1.27})^\circ$	
P8	$\vartheta_1 + \vartheta_3$	$(67.72^{+1.63}_{-1.46})^\circ$	$(66.93^{+1.68}_{-1.51})^\circ$	$(64.35^{+1.85}_{-1.64})^\circ$	$(61.16^{+2.02}_{-1.80})^\circ$	$(59.85^{+2.07}_{-1.84})^\circ$	$B_{>5}$
	ϑ_2	$(26.73^{+1.23}_{-1.15})^\circ$	$(28.24^{+1.13}_{-1.05})^\circ$	$(31.95^{+1.02}_{-0.94})^\circ$	$(34.90^{+1.07}_{-0.99})^\circ$	$(35.77^{+1.10}_{-1.01})^\circ$	
P9	$\vartheta_1 + \vartheta_3$	$(49.17^{+1.42}_{-1.36})^\circ$	$(49.62^{+1.42}_{-1.36})^\circ$	$(49.68^{+1.45}_{-1.37})^\circ$	$(48.25^{+1.53}_{-1.42})^\circ$	$(47.42^{+1.55}_{-1.43})^\circ$	B_4
	ϑ_2	$(41.01^{+1.35}_{-1.21})^\circ$	$(41.64^{+1.36}_{-1.21})^\circ$	$(43.43^{+1.27}_{-1.12})^\circ$	$(45.14^{+1.57}_{-1.40})^\circ$	$(45.70^{+1.64}_{-1.46})^\circ$	

nation of SC in the other eight schemes.

V. CP-VIOLATING PHASES

A. Analysis of results

The variation of lepton *CP*-violating phases in different schemes along with the variation of the *CP*-violating phase ϕ_3 is not trivial. From Fig. 1, the relationship between $\phi_1 \sim \phi_5$ and ϕ_3 is quite close to linear depen-

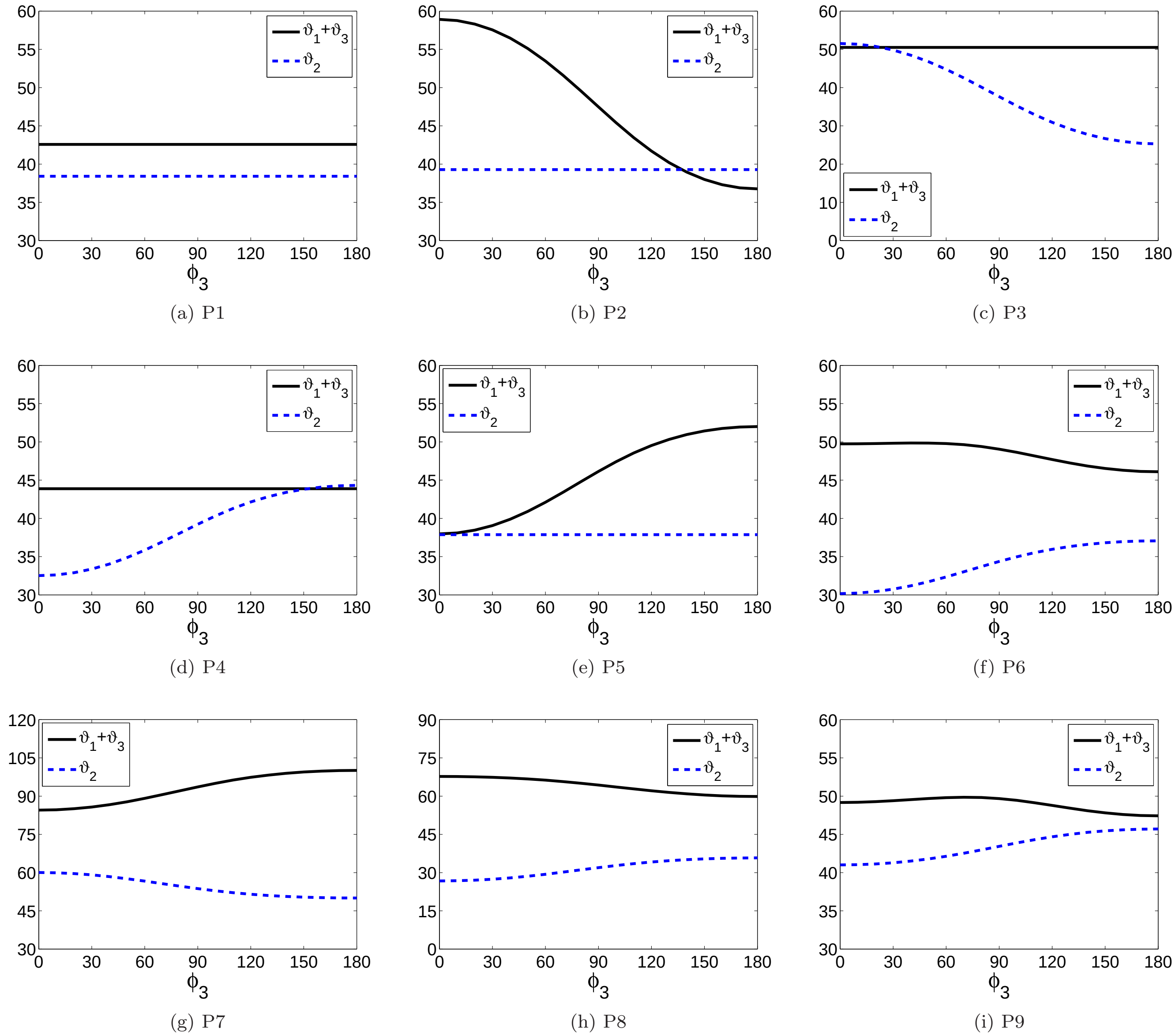


FIG. 3: The self-complementarity among lepton mixing angles [All the values are in the unit of degree (°).]

dence. More interesting are the nonmonotonous relations between $\phi_6 \sim \phi_9$ and ϕ_3. From Fig. 1 we guess that the correlation functions in P6∼P9 possess extremums with respect to ϕ_3, with the extremums reached when $\phi_3 \simeq 90°$. Actually, through calculations we know that the extremums in P6∼P9 are reached when ϕ_3 approaches approximately 87.0°, 78.4°, 92.0°, and 89.7°, respectively. Here the last one in P9 deserves attention, because it is quite close to 90°.

Furthermore, Fig. 1 implies a way of redefinition of *CP*-violating phases. By substituting $(180° - \phi)$ for the present *CP*-violating phases ϕ in P1, P8, and P9 schemes, we can unify P1∼P5 with the common property of similar quasilinear correlation functions between their *CP*-violating phases and ϕ_3. The others, P6∼P9 are also united in this way, holding the same property of the existence of a maximum value in similar correlation functions. This result also indicates that without any knowledge on lepton *CP*-violating phases, possible values of *CP*-violating phases in P6∼P9 schemes are already restricted by our known values of mixing angles. Meanwhile, possible values of *CP*-violating phases in P1∼P5 schemes are not restricted with current experimental data.

B. Maximal CP violation

It is necessary to clarify the meaning of "maximal *CP* violation" here. "Maximal *CP* violation" is defined as

the case when the magnitude of the scheme-independent Jarlskog invariant in the quark sector takes its maximal value. However, there are ambiguities on the choice of variables when we consider the meaning of "maximal." Originally, all the four parameters are viewed as variables, but such a maximized Jarlskog quantity is excluded in the quark sector experimentally. Now, it is prevalent to view only the *CP*-violating phase in each scheme as the variable with the mixing angles fixed. For instance, the analysis of maximal *CP* violation is usually carried out in the P3 scheme, where we regard mixing angles in P3 as constant and choose the *CP*-violating phase ϕ_3 as a variable. In this interpretation of "maximal *CP* violation," together with our previous analysis that *CP*-violating phases in P1∼P5 schemes are unrestricted, the meaning of maximal *CP* violation is, in fact, setting *CP*-violating phase in any one of P1∼P5 schemes to be 90°.

There is a conjecture that maximal *CP* violation is simultaneously satisfied in both the quark and the lepton sectors [24]. Taking the quark sector into consideration, from Table II we easily recognize that the P2 and P3 schemes in the quark sector possess large *CP*-violating phases that equal 90° within an error of 1σ, while in other schemes the *CP*-violating phases are far from 90°. Therefore, the P2 and P3 schemes are the favored ones when considering simultaneous maximal *CP* violation in both the quark and the lepton sector.

C. Empirical relations

Finally, some empirical relations of the quark *CP*-violating phases in different schemes are explored. To better illustrate the results, we use the *CP*-violating phase redefinition suggested above, i.e., substituting $(180° - \phi)$ for ϕ in P1, P8 and P9. For convenience, these nine *CP*-violating phases in the quark sector are relisted in Table V with our redefinitions. Some empirical relations we can easily read out are listed here:

$$\phi_1 \sim \phi_4 \sim \phi_5, \tag{32}$$
$$\phi_2 \sim \phi_3 \sim 90°, \tag{33}$$
$$\phi_6 \sim \phi_8 \sim \phi_9, \tag{34}$$
$$\phi_7 \sim 0°. \tag{35}$$

In fact, (32) are satisfied by the similarities between their mixing angles. From Table II, we have these relations approximately (here $\theta_{i(j)}$ represents θ_i in Pj scheme):

$$\theta_{12(1)} = \theta_{1(4)} = \theta_{1(5)}, \tag{36}$$
$$\theta_{23(1)} = \theta_{2(4)} = \theta_{2(5)}, \tag{37}$$
$$\theta_{13(1)} = \theta_{3(4)} = \theta_{3(5)}. \tag{38}$$

Then using the scheme-independent Jarlskog invariant, we get

$$1 = \frac{J_1}{J_4} = \frac{s_{12(1)}s_{23(1)}s_{13(1)}c_{12(1)}c_{23(1)}c_{13(1)}^2 \sin\phi_1}{s_{1(4)}s_{2(4)}s_{3(4)}c_{1(4)}^2 c_{2(4)}c_{3(4)} \sin\phi_4}$$
$$\simeq \frac{c_{13(1)}\sin\phi_1}{c_{1(4)}\sin\phi_4} \simeq \frac{\sin\phi_1}{\sin\phi_4}, \tag{39}$$
$$1 = \frac{J_4}{J_5} = \frac{s_{1(4)}s_{2(4)}s_{3(4)}c_{1(4)}^2 c_{2(4)}c_{3(4)} \sin\phi_4}{s_{1(5)}s_{2(5)}s_{3(5)}c_{1(5)}c_{2(5)}^2 c_{3(5)} \sin\phi_5}$$
$$\simeq \frac{c_{1(4)}\sin\phi_4}{c_{2(5)}\sin\phi_5} \simeq \frac{\sin\phi_4}{\sin\phi_5}, \tag{40}$$

justifying the relation (32). By the same way, (34) is justified through the similarities among mixing angles in P6, P8, and P9.

Relation (33) states possible maximal *CP* violation as we discussed before, and relation (35) merely reflects the relative largeness of the three mixing angles in P7. With the existence of *CP* violation confirmed, ϕ_7 cannot be exactly 0° though close to it [29].

We are willing to find out some similar empirical relations on lepton *CP*-violating phases. Unfortunately, similar relations cannot be easily found, because the lepton mixing angles are quite different from each other in different schemes, thus invalidating our method used for the quark sector.

VI. CONCLUSIONS

From the results of Sec. III and Sec. IV, the validation of QLC and SC depends on the choices of schemes and lepton *CP*-violating phases, and careful inspections should be carried out when we consider the generalization of QLC and SC from the standard CK scheme to the other eight schemes. On the issues of *CP*-violating phases, restrictions on lepton *CP*-violating phases in P6∼P9 are recognized. Simultaneous maximal *CP* violation in both the quark and the lepton sector is possible in the P2 and P3 scheme. A redefinition of *CP*-violating phases for unification is suggested and some empirical relations on the quark *CP*-violating phases are explored. All of these results may enrich our knowledge of QLC, SC relations, and *CP*-violating phases, helping us understand the mystery of lepton mixing.

Acknowledgments

This work is supported by the Principal Fund for Undergraduate Research at Peking University. It is also partially supported by the National Natural Science Foundation of China (Grants No. 11021092, No. 10975003, No. 11035003, and No. 11120101004), by the National Fund for Fostering Talents of Basic Science (Grants No. J1030310), and by the Research Fund for the Doctoral Program of Higher Education (China).

TABLE V: The quark *CP*-violating phases in nine schemes

ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8	ϕ_9
$\left(110.90^{+3.85}_{-2.02}\right)^\circ$	$\left(89.69^{+2.29}_{-3.95}\right)^\circ$	$\left(89.29^{+3.99}_{-2.33}\right)^\circ$	$\left(111.95^{+3.82}_{-2.02}\right)^\circ$	$\left(111.94^{+3.85}_{-2.02}\right)^\circ$	$\left(22.72^{+1.25}_{-1.18}\right)^\circ$	$\left(1.08^{+0.06}_{-0.06}\right)^\circ$	$\left(22.69^{+1.25}_{-1.18}\right)^\circ$	$\left(21.68^{+1.20}_{-1.13}\right)^\circ$

[1] B. Pontecorvo, Sov. Phys. JETP **26**, 984 (1968); Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[3] L.L. Chau and W.Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984).

[4] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** 652 (1973).

[5] Y. Koide, Phys. Lett. B **607**, 123 (2005).

[6] Y. Koide and H. Nishiura, Phys. Rev. D **79**, 093005 (2009).

[7] P.H. Frampton and X.-G. He, Phys. Lett. B **688**, 67 (2010); P.H. Frampton and X.-G. He, Phys. Rev. D **82**, 017301 (2010).

[8] S.W. Li and B.-Q. Ma, Phys. Lett. B **691**, 37 (2010).

[9] N. Qin and B.-Q. Ma, Phys. Lett. B 695, 194 (2011); N. Qin and B.-Q. Ma, Phys. Rev. D 83, 033006 (2011).

[10] Y.H. Ahn, H.Y. Cheng and S. Oh, Phys. Lett. B **701**, 614 (2011).

[11] H. Fritzsch and Z.-Z. Xing, Phys. Rev. D **57** 594 (1998).

[12] Y.-j. Zheng, Phys. Rev. D **81**, 073009 (2010).

[13] X. Zhang and B.-Q. Ma, Phys. Lett. B **710**, 630 (2012).

[14] X. Zhang, Y.-j. Zheng and B.-Q. Ma, Phys. Rev. D **85** 097301 (2012).

[15] A. Y. Smirnov, arXiv:hep-ph/0402264.

[16] H. Minakata and A.Y. Smirnov, Phys. Rev. **70**, 073009 (2004).

[17] M. Raidal, Phys. Rev. Lett. **93**, 161801 (2004).

[18] See, e.g., P. H. Frampton and R. N. Mohapatra, J. High Energy Phys. **0501**, 025 (2005); N. Li and B.-Q. Ma, Phys. Rev. D **71**, 097301 (2005); S. Antusch, S. F. King and R. N. Mohapatra, Phys. Lett. B **618**, 150 (2005); H. Minakata,; J. Ferrandis and S. Pakvasa, Phys. Rev. D **71**, 033004 (2005); S.K. Kang, C.S. Kim and J. Lee, Phys. Lett. B **619**, 129 (2005); M.A. Schmidt and A.Y. Smirnov, Phys. Rev. D **74**, 113003 (2006); K. A. Hochmuth and W. Rodejohann, Phys. Rev. D **75**, 073001 (2007); F. Plentinger, G. Seidl and W. Winter, Phys. Rev. D **76**, 113003 (2007); G. Altarelli, F. Feruglio and L. Merlo, J. High Energy Phys. **0905**, 020 (2009).

[19] Y.-j. Zheng and B.-Q. Ma, Eur. Phys. J. Plus **127**, 7 (2012).

[20] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[21] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).

[22] C. Jarlskog, Phys. Rev. Lett. **55** 1039 (1985); D.-d. Wu, Phys. Rev. D **33** 860 (1986); O.W. Greenberg, Phys. Rev. D **32** 1841 (1985).

[23] G. L. Fogli, E. Lisi, A. Marrone, D. Montanino, A. Palazzo, and A. Rotunno, Phys. Rev. D **86**, 013012 (2012).

[24] X. Zhang and B.-Q. Ma, Phys. Lett. B **713**, 202 (2012); arXiv:1204.6604 [hep-ph].

[25] K. Abe *et al.*, Phys. Rev. Lett. **107** 041801 (2011),.

[26] P.Adamson *et al.*, Phys. Rev. Lett. **107**, 181802 (2011).

[27] C. Jarlskog Phys. Lett. B **625**, 63 (2005).

[28] G.-N. Li, H.-H. Lin and X.-G. He, Phys. Lett. B **711**, 57 (2012).

[29] See, also, Jean-Marc Gerard, arXiv:0811.0540.